

Violation of Bell's inequalities with amplified entangled coherent states

G. Torlai^{1,2}, G. McKeown², P. Marek³, R. Filip³, H. Jeong⁴, M. Paternostro², and G. De Chiara²

¹*Department of Physics, Ludwig Maximilians Universität, Schellingstraße 4 80799, Munich, Germany*

²*Centre for Theoretical Atomic, Molecular and Optical Physics,*

School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom

³*Department of Optics, Palacký University, 17. listopadu 1192/12, 77207 Olomouc, Czech Republic*

⁴*Center for Macroscopic Quantum Control, Department of Physics and Astronomy,
Seoul National University, Seoul, 151-742, Korea*

(Dated: January 16, 2013)

We show that the use of probabilistic noiseless amplification in entangled coherent state-based schemes for the test of quantum non locality provides significant advantages. The threshold amplitude to falsify a Bell-CHSH non locality test, in fact, is significantly reduced when amplification is embedded into the test itself, while any detection-inefficiency effect can be effectively compensated by tuning the amplification gain. We extend our analysis to general tests of quantum non locality in phase space, therefore affirming noiseless amplification as a valuable tool for coherent information processing and the generation of strongly non-classical states of bosonic systems.

It is well known that entangled two-mode states endowed with a Gaussian Wigner function [1] and subjected to Gaussian phase-space measurements are unable to reveal any nonlocal feature. This point was originally used by Bell to conjecture that the (non-normalized) entangled Einstein-Podolski-Rosen state $\int_{-\infty}^{\infty} dx |x, x + x_0\rangle$ [2] (with $|x\rangle$ and $|x + x_0\rangle$ two position eigenstates of a harmonic oscillator), whose Wigner function is positive in the whole phase space, would not falsify any local hidden variable model [3]. However, Banaszek and Wódkiewicz later devised a phase-space approach based on the statistics gathered from the measurement of photon parity operators [4], demonstrating the key role played by non-Gaussianity in the revelation of the nonlocal feature of entangled two-mode states. This sort of approach finds its complement in nonlocality tests performed using Gaussian operations and measurements on non-Gaussian states, such as entangled coherent states (ECS) [5] or de-Gaussified two-mode states achieved by resorting to photon-subtraction (photon-addition) [6–10].

In particular, the nonlocal nature of an ECS has been extensively studied in the past years, addressing tests based on effective pseudo-spin operators, photon parity operators, effective rotations, and dichotomized homodyne measurements, even in the presence of decoherence [11–15]. The latter approaches have been used for the violation of Bell-CHSH inequality [16] by states having a very large thermal occupation number [17], thus showing the possibility to reveal their nonclassical character even under mechanisms that, naively, would be expected to wash out any quantumness. A conspicuous feature of ECS-based tests using homodyne measurements is that the violation of a Bell-CHSH inequality occurs only for coherent-state components having amplitude larger than a given threshold. Under realistic conditions, the threshold is typically determined by the operative conditions (detection inefficiencies and purity of the state resource, among other factors) under which the test is run. In light of the experimental difficulties encountered in the generation of ECS of large-amplitude components [18], it

is clearly desirable to identify viable strategies for the falsification of local realistic theories with lower amplitude thresholds, so as to ease the experimental efforts required for such an important task.

In this paper we report a test of local realism for ECS of light having an arbitrarily small amplitude, supplemented by the application of local heralded noiseless amplification to the components of the system [19, 20]. By increasing the amplitude of a coherent-state components without amplifying the quantum fluctuations, we probabilistically prepare a state which can approach maximal violation of the Bell inequalities. The threshold for the violation of the CHSH inequality can be considerably lowered, thus realising the mechanism sought above and increasing the resilience of the state resource to key sources of imperfections, such as inefficient measuring apparatuses.

The remainder of this paper is organised as follows. In Sec. I we gradually introduce the effects of local amplification on the protocol for the violation of Bell-CHSH inequalities with ECS, local rotations, and dichotomic homodyne measurements. We first address the non-physical case of ideal noiseless amplification, providing the rationale for our proposal. We show that the threshold value of the coherent-state amplitudes for the violation of a Bell-CHSH inequality decreases with the amplification gain. We then turn to an experimentally implementable approximation of the full amplifier, demonstrating that the predicted effect persists even at the lowest significant order (with respect to the gain) in the series-expansion of the amplification operator. Our efforts to put our proposal as close as possible to experimental capabilities are then pushed, in Sec. II, to comprise the influences of inefficient homodyne detection and the modification to the behavior of the Bell-CHSH function induced by the use of a series of physical operations that, for coherent states of large amplitudes, approximate well the effects of the local rotations. We show that the amplification is effective in reducing the threshold amplitude even under such unfavourable conditions, up to the point that the detection

inefficiencies (effects of approximated local rotations) can be fully counteracted so as to reinstate performances very close to those corresponding to a fully ideal situation. Finally, in Sec. III, we draw our conclusions and provide an outlook for future developments along the lines of this paper.

Our work strengthens the role of noiseless local amplification in coherent quantum information processing, showing its usefulness in the design of tests for the revelation of non classicality in important classes of entangled states.

I. BELL-CHSH INEQUALITY WITH LOCALLY AMPLIFIED ECS

A. Full amplification

We consider the unnormalised ECS $|\text{ECS}_+(\alpha)\rangle = |\alpha, \alpha\rangle + |-\alpha, -\alpha\rangle$ with $|\alpha\rangle$ a coherent state of amplitude $\alpha \in \mathbb{C}$. It is well-known that for even moderately large values of α , we have $\langle \alpha | -\alpha \rangle \simeq 0$, which entails the fact that, upon proper normalization, $|\text{ECS}_+(\alpha)\rangle$ carries up to a full ebit of entanglement for $\alpha \gtrsim 1$. On the other hand, for $\alpha \ll 1$ the state approaches the unnormalized state $|00\rangle + \alpha^2|11\rangle$ in the space spanned by the Fock states $\{|0\rangle, |1\rangle\}$, which can also violate a Bell inequality despite its weak degree of entanglement. However, the entanglement is quite particle-type, due to the single-excitation Fock state-based decomposition above.

Following the proposal put forward in Ref. [14], the nonlocal nature of ECSs can be tested by means of local operations, implemented by cascading linear and non-linear transformations, and dichotomized homodyne measurements. We modify such earlier scheme by introducing, immediately, the key point of our protocol, which consists of supplementing such local transformations with local amplification stages, along the lines of the scheme shown in Fig. 1. We thus introduce the local

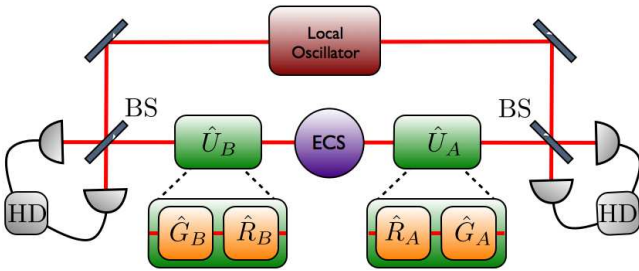


FIG. 1: Scheme for the violation of the CHSH inequality with amplified entangled coherent state. We show the source of ECS states, the local oscillator (LO) needed for homodyne measurements, and the decomposition of the local unitary transformations \hat{U}_j given in terms of the rotations $\hat{R}(\theta_j)$ and local amplification \hat{G}_j ($j = A, B$). We also show the symbols for beamsplitters and homodyners.

transformations

$$\hat{U}_j = \hat{G}_j \hat{R}_j(\theta_j) \quad (j = A, B) \quad (1)$$

with $\hat{R}_j(\theta_j)$ the local rotations in the space spanned by the quasi-orthogonal coherent states $\{|\alpha\rangle, |-\alpha\rangle\}$ that have been first discussed in [14] and whose form reads

$$\hat{R}_j(\theta_j)\mathbf{v}_j = \begin{pmatrix} \cos \theta_j & \sin \theta_j \\ \sin \theta_j & -\cos \theta_j \end{pmatrix} \mathbf{v}_j. \quad (j = A, B) \quad (2)$$

Here, $\mathbf{v}_j = (|\alpha\rangle_j, |-\alpha\rangle_j)^T$ is the vector of coherent-state components for mode j . The other transformation in our scheme is the local noiseless amplification described by the operator $\hat{G}_j = \exp[(g-1)\hat{a}_j^\dagger \hat{a}_j]$ [21], where $g \geq 1$ is the gain of the amplifier and \hat{a}_j and \hat{a}_j^\dagger are the bosonic annihilation and creation operators for mode j . For now, we retain the full form of the amplification operator to illustrate, in a clear-cut way, the working principle of our proposal.

Let us consider the effect of $\hat{U}_A \otimes \hat{U}_B$ on the ECS $|\text{ECS}_+(\alpha)\rangle$. As $\hat{G}_j |\alpha\rangle_j = |\tilde{\alpha}\rangle_j$ with $\tilde{\alpha} = \alpha e^{g-1}$, it is straightforward to show that

$$\begin{aligned} |\psi_f\rangle &= \mathcal{N}(\hat{U}_A \otimes \hat{U}_B) |\text{ECS}_+(\alpha)\rangle \\ &= \mathcal{N} \{ \cos[2(\theta_B - \theta_A)] |\text{ECS}_+(\tilde{\alpha})\rangle \\ &\quad + \sin[2(\theta_B - \theta_A)] |\text{ECS}'_-(\tilde{\alpha})\rangle \} \end{aligned} \quad (3)$$

with \mathcal{N} the normalization factor

$$\mathcal{N} = \left(2 + 2\nu e^{-4\tilde{\alpha}^2} \right)^{-1/2}, \quad (4)$$

$\nu = \cos[2(\theta_A - \theta_B)]$, and where we have introduced the unnormalized ECS $|\text{ECS}'_-(\alpha)\rangle = |\alpha, -\alpha\rangle_{AB} - |-\alpha, \alpha\rangle_{AB}$. Eq. (3) has the very same structure that would be taken by $|\text{ECS}_+(\alpha)\rangle$ upon bi-local rotation and no amplification [14], the only change being the actual amplitude of the coherent-state components. In turn, this implies that, upon application of the proposal for Bell-CHSH test discussed in [14, 15], which is based on dichotomized homodyne measurements performed on modes A and B , we get the following expression for the correlation function between measurement outcomes following the rotation of the modes' state by θ_A and θ_B respectively

$$\mathcal{C}(\tilde{\alpha}, \theta_A, \theta_B) = \frac{\text{Erf}^2[\sqrt{2}\tilde{\alpha}]\nu}{1 + \nu e^{-4\tilde{\alpha}^2}} \quad (5)$$

with $\text{Erf}[\cdot]$ the Error function. In this framework, the Bell-CHSH function is written as

$$\begin{aligned} \mathcal{B}(\tilde{\alpha}, \Theta) &= \mathcal{C}(\tilde{\alpha}, \theta_{A1}, \theta_{B1}) + \mathcal{C}(\tilde{\alpha}, \theta_{A1}, \theta_{B2}) \\ &\quad + \mathcal{C}(\tilde{\alpha}, \theta_{A2}, \theta_{B1}) - \mathcal{C}(\tilde{\alpha}, \theta_{A2}, \theta_{B2}), \end{aligned} \quad (6)$$

where $\Theta = \{\theta_{A1}, \theta_{A2}, \theta_{B1}, \theta_{B2}\}$ is a set of rotations angles. Local realistic theories impose the bound $|\mathcal{B}| \leq 2$. Quantum mechanically, this inequality can be violated using ECSs, the set of rotations in Eq. (2) and dichotomic

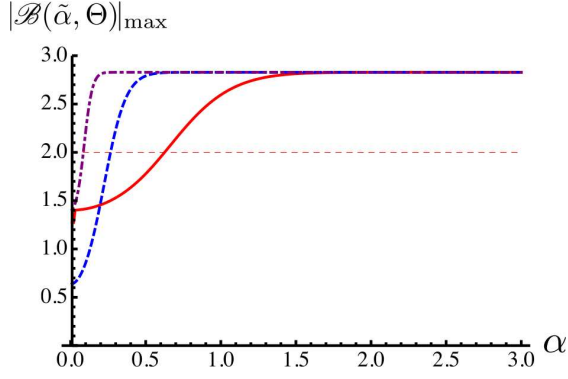


FIG. 2: Bell-CHSH function $\mathcal{B}(\tilde{\alpha}, \Theta)$, optimized over the set of rotation angles Θ , plotted against α for $g = 1$ (solid red line), $g = 2$ (blue dashed line) and $g = 3$ (purple dot-dashed one). The light straight line marks the local realistic bound.

homodyne detection. From this analysis it is clear that, by calling $\bar{\alpha}$ the amplitude of the coherent-state components at which the Bell-CHSH inequality is first violated and having prepared $|\text{ECS}_+(\alpha_a)\rangle$ with $\alpha_a \ll \bar{\alpha}$, we can get $\mathcal{B} > 2$ using an appropriate gain, according to the relation

$$g \geq 1 + \ln(\bar{\alpha}/\alpha_a). \quad (7)$$

The behavior of \mathcal{B} against α and for a set of values of the gain is shown in Fig. 2, which demonstrates the quick saturation of the Bell-CHSH function to the Csirel'son bound $2\sqrt{2}$ and the reduction (exponential with the value of the gain g) in the threshold amplitude for the violation of the inequality. Finally, let us notice that the ordering of the rotation and amplification operations is actually not important and the very same violation of the Bell-CHSH inequality would be obtained by considering the unitary operator $\hat{U}'_A \otimes \hat{U}'_B$ with $\hat{U}'_j = \hat{R}_j(\theta_j)\hat{G}_j$.

B. Effective amplification

It is well known that the unbound nature of \hat{G}_j makes the transformation $|\alpha\rangle \rightarrow |\alpha e^{g-1}\rangle$ unphysical and implementable only probabilistically. The realization of noiseless amplification has been at the centre of an intense theoretical and experimental activity aiming at proposing and implementing effective noiseless amplification [19–24]. For weak coherent states and small values of the gain, the amplification operator can be expanded to the first order in g as [19]

$$\hat{G}_j \simeq \hat{\mathbb{1}} + (g-1)\hat{a}_j^\dagger \hat{a}_j = (g-2)\hat{a}_j^\dagger \hat{a}_j + \hat{a}_j \hat{a}_j^\dagger. \quad (8)$$

The amplification thus results in the application of a weighted coherent superposition of the operators $\hat{a}_j^\dagger \hat{a}_j$ and $\hat{a}_j \hat{a}_j^\dagger$. Both photon-subtraction and addition operations have already been realized experimentally for arbitrary states of light [25]. A general superposition of

these two operators can be experimentally engineered with a suitable configuration of stimulated parametric down-conversion and linear optics elements and with only a negligible contribution from multiphoton events [26].

A remark is due at this stage: contrary to the case of full amplification operators, when Eq. (8) is used, the actual ordering of the amplification and rotation stages is key to the success of the overall protocol. In particular, it takes a straightforward albeit lengthy calculation to show that, when the amplification (with $g \ll 1$) precedes the bilocal rotations, no advantage with respect to the no-amplification version of the scheme is achieved. Indeed, the state resulting from the application of the operator $\hat{U}'_A \otimes \hat{U}'_B$ [with \hat{G}_j approximated as in Eq. (8)] reads

$$\begin{aligned} |\psi_f\rangle &= \mathcal{N}'(\hat{U}'_A \otimes \hat{U}'_B) |\text{ECS}_+(\alpha)\rangle \\ &\simeq \mathcal{N}[\cos[2(\theta_B - \theta_A)] |\text{ECS}_+(\alpha)\rangle \\ &\quad + \sin[2(\theta_B - \theta_A)] |\text{ECS}'_-(\alpha)\rangle], \end{aligned} \quad (9)$$

which bears no dependence on the amplification gain. Differently, we will prove in what follows that amplification following local rotations indeed results in a more advantageous resource that exhibits features similar to those of the fully amplified state in Eq. (3). We thus describe the protocol for the construction of the Bell-CHSH function resulting from the application of the \hat{U}_j 's onto $|\text{ECS}_+(\alpha)\rangle$ and dichotomized homodyne measurements. This demonstrates that noiseless amplification is important to fulfill the demanding task at the core of this paper.

The initial state $|\text{ECS}_+(\alpha)\rangle$ is correspondingly transformed into $|\psi_f\rangle = (\hat{U}_A \otimes \hat{U}_B) |\text{ECS}_+(\alpha)\rangle$ and measured via homodyne detection. Taking $\alpha \in \mathbb{R}$ without loss of generality, the joint probability amplitude to get homodyne signals x_A and x_B at site A and B respectively is

$$C_g(x_A, x_B, \theta_A, \theta_B) = \sum_{\gamma=\pm\alpha} \Gamma_\gamma^g(x_A, \theta_A) \Gamma_\gamma^g(x_B, \theta_B), \quad (10)$$

where $\Gamma_{\pm\alpha}^g(x_j, \theta_j) = {}_j\langle x | \hat{U}_j | \pm\alpha \rangle_j$ and $|x\rangle_j$ is an eigenstate of the quadrature operator $\hat{x}_j = (\hat{a}_j^\dagger + \hat{a}_j)/2$. An explicit calculation gives us

$$\Gamma_{\pm\alpha}^g(x_j, \theta_j) = \frac{1}{\sqrt[4]{\pi}} [\xi_{\mp\alpha}(x_j) \sin \theta_j \pm \xi_{\pm\alpha}(x_j) \cos \theta_j], \quad (11)$$

where we have introduced the functions $\xi_{\pm\alpha}(y) = e^{-(y \mp \alpha)^2} [1 + (g-1)(\pm 2\alpha y - \alpha^2)]$ ($y = x_A, x_B$). To construct the Bell function the continuous variables must be dichotomized. This is done by assigning a value $+1$ to an homodyne measurement larger than 0 and -1 otherwise, constructing in this way a set of dichotomic observables. The joint probabilities of the measurement outcomes are

$$P_{kl}^g(\theta_A, \theta_B) = \frac{1}{K} \int_{\Omega_k} dx_A \int_{\Omega_l} dx_B |C_g(x_A, x_B, \theta_A, \theta_B)|^2, \quad (12)$$

where $k, l = \pm$ correspond to the bilateral measurement outcomes ± 1 , $\Omega_+ = [0, \infty]$, $\Omega_- = [-\infty, 0]$ and K is a normalization constant. The Bell-CHSH function is then

$$B^g(\alpha, \Theta) = \mathcal{C}^g(\alpha, \theta_{A1}, \theta_{B1}) + \mathcal{C}^g(\alpha, \theta_{A1}, \theta_{B2}) + \mathcal{C}^g(\alpha, \theta_{A2}, \theta_{B1}) - \mathcal{C}^g(\alpha, \theta_{A2}, \theta_{B2}), \quad (13)$$

with the correlation function

$$\begin{aligned} \mathcal{C}^g(\alpha, \theta_A, \theta_B) &= \sum_{k=\pm} P_{kk}^g(\theta_A, \theta_B) - \sum_{k \neq l=\pm} P_{kl}^g(\theta_A, \theta_B) \\ &= \frac{\sqrt{\mu_\alpha} \nu \text{Erf}[\sqrt{2}\alpha]}{\sqrt{\pi}(\mu_\alpha + \nu)^2} \left\{ 4\sqrt{2}\alpha(g-1)(\mu_\alpha + \nu) + \sqrt{\pi\mu_\alpha} \text{Erf}[\sqrt{2}\alpha] \right. \\ &\quad \left. \times [\mu_\alpha + (1 + 8(g-1)\alpha^2)\nu] \right\} \end{aligned} \quad (14)$$

and $\mu_\alpha = \exp[4\alpha^2]$. While a local realistic description of the entangled coherent state in the presence of the ideal local rotations and without amplification is not possible for $\alpha \gtrsim 0.63$, for a state locally amplified by $g = 1.1$ such threshold is lowered to 0.57 [cf. inset of Fig. 3].

Further reductions of the threshold value of α can be obtained increasing the gain, still remaining within the limits of validity of the second-order expansion within which our calculations have been performed. For instance, in the main panel of Fig. 3 we show the Bell function, optimised numerically over Θ , for $g = 1$ (red curve) and $g = 1.4$ (blue curve), plotted against the coherent-state amplitude α . The value of α at which the Bell-CHSH inequality is first violated when the state is locally amplified goes down to 0.43, an approximately 30% reduction in the value corresponding to no local amplification. In this case the inaccuracy due to the second order expansion in g is about 2×10^{-3} . As an example, we report the value of the optimized Bell's function without amplification for $\alpha = 0.7$ which is $B_{id}^1(0.7, \Theta_{0.7}) \simeq 2.14$, and compare it to $B_{id}^{1.4}(0.7, \Theta_{0.7}) \simeq 2.76$, which corresponds to $g = 1.4$. We can see that, already at $\alpha = 0.7$, the Bell's function is almost saturated.

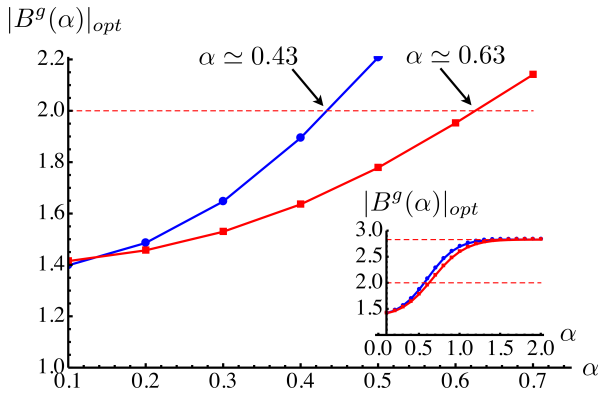


FIG. 3: Bell function (optimized numerically over the set of rotation angles Θ) plotted against the coherent-state amplitude α for $g = 1.0$ (red curve) and $g = 1.1$ (blue curve). Inset: same as in the main panel but for the extended range of values of $\alpha \in [0, 2]$.

II. INEFFICIENT HOMODYNE DETECTORS AND EFFECTIVE ROTATIONS

In this Section we show the effect that inefficient detectors have on the behavior of the Bell function. Moreover, we replace the idealized local rotations in Eq. (2) with a cascade of local unitary operations whose resulting effect on a single mode is to approximate $\hat{R}_j(\theta_j)$. As shown in Refs. [14, 15], both the detection inefficiency and the replacement of the idealized rotations with effective ones increase the threshold value of α for the violation of the Bell-CHSH inequality. In what follows we show that the implementation of local amplification stages can compensate fully these features.

Let us start with the analysis of non-ideal homodyne detectors, each being modelled as a perfect detector preceded by beam splitters of transmittivity η . The latter mixes mode j to an ancillary mode \hat{a}_j ($j = A, B$) prepared in the vacuum state. At the output port of the beam splitter, the reduced state of mode j (after tracing out the corresponding ancilla) describes the signal detected by a homodyner of efficiency η .

By proceeding along the lines of the calculations described in Sec. I, we get the correlation function

$$\begin{aligned} \mathcal{C}_d^g(\alpha, \theta_A, \theta_B) &= \frac{\mu_\alpha \nu e^{-2\kappa_\eta^2} \text{Erf}(\sqrt{2}\kappa_\eta)}{[\sqrt{\pi}(\mu_\alpha + \nu)^2]} \left\{ 4\sqrt{2}\kappa_\eta(\mu_\alpha + \nu) \right. \\ &\quad \left. + \sqrt{\pi}e^{2\kappa_\eta^2} [\mu_\alpha + [1 + 2(g-1)(4\alpha^2 + \kappa_\eta^2)]\nu \text{Erf}(\sqrt{2}\kappa_\eta)] \right\} \end{aligned}$$

with $\kappa_\eta = \eta\alpha$. In Fig. 4 we compare the optimized Bell function for no gain and detection efficiency $\eta = 0.9$ to what is obtained by introducing the local amplification stages (with $g = 1.4$) and for the same value of η . Not only the amplified ECS violates the Bell-CHSH inequality for smaller values of α than the non-amplified state affected by the same degree of detection inefficiency: it also overcomes the performance of the Bell function for no amplification and ideal homodyne detectors, thus demonstrating the effectiveness of this strategy to counteract non-idealities at the detection stage.

We now pass to the construction of the correlation function resulting from the use of the operations approximating the local rotation operators on each mode of our system. In order to simplify our mathematical approach, from here on we will only consider ideal detectors, the extension to imperfect ones being performed following the lines sketched above. Eq. (2) is well approximated by the cascade of the transformation resulting from the self-Kerr Hamiltonian $\hat{H}_j = \hbar\Omega(\hat{a}_j^\dagger \hat{a}_j)^2$ and a phase-space displacement by an appropriate amplitude according to the overall expression

$$\hat{V}_j(\theta_j) = e^{i\pi(\hat{a}_j^\dagger \hat{a}_j)^2} \hat{D}_j(i\theta_j/\alpha) e^{i\pi(\hat{a}_j^\dagger \hat{a}_j)^2}. \quad (15)$$

When applied to the components of the set of quasi-orthogonal states $\{|\alpha\rangle_j, |-\alpha\rangle_j\}$, this leads to the follow-

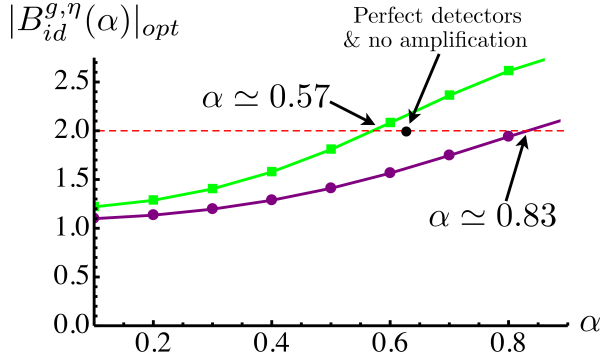


FIG. 4: Numerically optimized Bell's function plotted against the amplitude of the coherent states with detection inefficiencies. The black point indicates the value of α for which the violation occurs with perfect detectors ($\eta = 1$) and no amplification. Setting $\eta = 0.9$ we obtained the purple curve for $g = 1.0$ and the green curve for $g = 1.4$

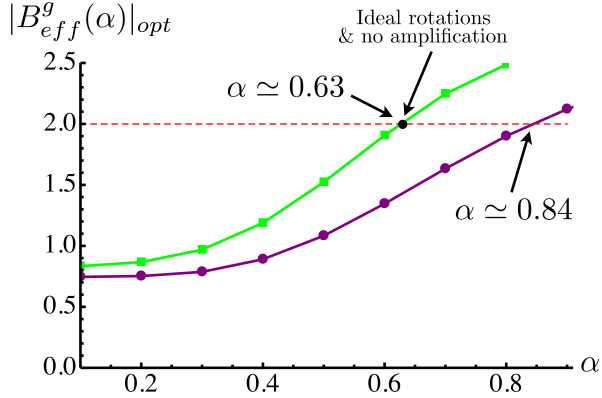


FIG. 5: Numerically optimized Bell's function plotted against the amplitude of the coherent states with effective rotations for $g = 1.0$ (purple curve) and $g = 1.3$ (green curve). The black point represents the value of α for which the violation occurs with ideal rotations and no amplification

ing set of transformations [14]

$$\begin{aligned}\hat{V}_j(\theta_j) |\alpha\rangle_j &= \frac{1}{2} [e^{i\theta_j}(|\alpha + i\theta_j/\alpha\rangle_j + i|-\alpha - i\theta_j/\alpha\rangle_j) \\ &\quad + ie^{i\theta_j}(|-\alpha + i\theta_j/\alpha\rangle_j + i|\alpha - i\theta_j/\alpha\rangle_j)] \\ \hat{V}_j(\theta_j) |-\alpha\rangle_j &= \frac{1}{2} [ie^{i\theta_j}(|\alpha + i\theta_j/\alpha\rangle_j + i|-\alpha - i\theta_j/\alpha\rangle_j) \\ &\quad + e^{i\theta_j}(|-\alpha + i\theta_j/\alpha\rangle_j + i|\alpha - i\theta_j/\alpha\rangle_j)].\end{aligned}\quad (16)$$

In order to evaluate the correlation functions upon local rotations and homodyne detection, we replace Eq. (10) with

$$C_g^{\text{eff}}(x_A, x_B, \theta_A, \theta_B) = \sum_{\gamma=\pm\alpha} \Pi_\gamma^g(x_A, \theta_A) \Pi_\gamma^g(x_B, \theta_B) \quad (17)$$

with $\Pi_{\pm\alpha}^g(x_j, \theta_j) = {}_j\langle x_j | \hat{G}_j \hat{V}_j(\theta_j) | \pm\alpha \rangle_j$. We get

$$\begin{aligned}\Pi_{\pm\alpha}^g(x_j, \theta_j) &= \mp \frac{i^{\delta_{\pm\alpha}^j}}{\sqrt{\pi}} \left[ie^{i\theta_j} (\xi_{\chi_+}^+(x_j, \theta_j) + i\xi_{\chi_+}^-(x_j, \theta_j)) \right. \\ &\quad \left. \mp e^{-i\theta_j} (\xi_{\chi_-}^-(x_j, \theta_j) + i\xi_{\chi_-}^+(x_j, \theta_j)) \right],\end{aligned}\quad (18)$$

where we have introduced $\chi_{\pm}^j = \alpha \pm \frac{i\theta_j}{\alpha}$ and

$$\xi_{\chi_{\pm}}^{\pm}(x_j, \theta_j) = e^{-(x_j \mp \chi_{\pm}^j)^2} [1 + (g-1)(\pm 2\chi_{\pm}^j x_j - \chi_{\pm}^{j2})]. \quad (19)$$

Fig. 5 shows the optimized Bell's function with effective rotations for $g = 1.0$ (purple curve) and $g = 1.3$ (green curve). In this case, the threshold for the violation of the Bell-CHSH inequality is lowered from $\alpha = 0.84$, which is the value achieved using the effective rotations, to $\alpha = 0.63$, corresponding to the use of the ideal rotation, perfect homodyne measurements, and no amplification. Clearly, the use of locally amplified ECSs would enhance the performance of this class of states up to the point that any imperfection arising at the local-unitary and detection level can be counteracted effectively.

III. CONCLUSIONS AND OUTLOOK

We have demonstrated the effectiveness of local noiseless amplification in lowering the threshold for the violation of a Bell-CHSH inequality by an ECS. The strategy that we have applied consists of local rotations performed over the two modes of the system and dichotomic homodyne measurements, which are known to be effective in revealing the nonlocal properties of ECSs. The advantages of using local amplifiers is evident in a significant reduction of the amplitude of the coherent-state components of the ECS needed to go beyond the bound imposed by local realistic theories. Such effects are strong enough that arbitrary detection inefficiencies can be counteracted so as to reproduce performances typical of an ideal protocol, and thus demonstrate the efficiency of our strategy. It will be very interesting to extend the domain of usefulness of local noiseless amplification for quantum information processing by addressing the violation of a Bell-CHSH inequality through local photon parity measurements performed over entangled Gaussian states, such as a two-mode squeezed vacuum state. Our task is to affirm approximate noiseless amplification as a valid and viable alternative to the use of conditional photo-subtraction for the enhancement of the non-locality properties of interesting classes of continuous-variable states.

Acknowledgments

GT and GMcK thank the Centre for Theoretical Atomic, Molecular, and Optical Physics, Queen's Univer-

sity Belfast, and the Department of Optics, Palacký University, respectively, for hospitality during various stages of this work. We acknowledge financial support from the UK EPSRC through a Career Acceleration Fellowship and the “New Directions for EPSRC Research Lead-

ers” initiative (EP/G004759/1) as well as the National Research Foundation of Korea (NRF) grant funded by the Korean Government (No. 2010-0018295). RF and PM acknowledge projects P205/12/0577 and Post-doc project P205/10/P319 of GA ČR, respectively.

-
- [1] E. P. Wigner, Phys. Rev. **40**, 749 (1932).
 - [2] A. Einstein, B. Podolski, and N. Rosen, Phys. Rev. **47**, 777 (1935).
 - [3] J. S. Bell, *Speakable and Unsayable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).
 - [4] K. Banaszek, and K. Wódkiewicz, Phys. Rev. Lett. **82**, 2009 (1999).
 - [5] B. C. Sanders, Phys. Rev. A **45**, 6811 (1992); J. Phys. A **45**, 244002 (2012).
 - [6] S. Olivares, and M. G. A. Paris, Phys. Rev. A **70**, 032112 (2004); J. Opt. B: Quantum Semiclass. Opt. **7**, S392 (2005).
 - [7] R. Garcia-Patron Sanchez, J. Fiurášek, N. J. Cerf, J. Wenger, R. Tualle-Brouri, and Ph. Grangier, Phys. Rev. Lett. **93**, 130409 (2004).
 - [8] H. Jeong, Phys. Rev. A **78**, 042101 (2008).
 - [9] H. Nha and H. Carmichael, Phys. Rev. Lett. **93**, 020401 (2004).
 - [10] J. Park, S.-Y. Lee, H.-W. Lee, and H. Nha, J. Opt. Soc. Am. B **29**, 906 (2012).
 - [11] D. Wilson, H. Jeong, and M. S. Kim, J. Mod. Opt. **49**, 851 (2002).
 - [12] H. Jeong, W. Son, M. S. Kim, J. Lee, and Č. Brukner, Phys. Rev. A **67**, 012106 (2003).
 - [13] R. Filip, J. Rehacek, and M. Dusek, J. Opt. B: Quantum Semiclass. Opt. **3**, 341 (2001).
 - [14] M. Stobińska, H. Jeong, and T. C. Ralph, Phys. Rev. A **75**, 052105 (2007).
 - [15] H. Jeong, M. Paternostro, and T. C. Ralph, Phys. Rev. Lett. **102**, 060403 (2009).
 - [16] J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
 - [17] R. Filip, M. Dusek, J. Fiurasek, and L. Mista, Phys. Rev. A **65**, 043802 (2002); H. Jeong, and T. C. Ralph, Phys. Rev. Lett. **97**, 100401 (2006).
 - [18] A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri and P. Grangier, Nature **448**, 784 (2007).
 - [19] A. Zavatta, J. Fiurášek, and M. Bellini, Nature Photon. **5**, 52 (2011).
 - [20] G. Y. Xiang, T. C. Ralph, A. P. Lund, N. Walk, and G. J. Pryde, Nature Photon. **4**, 316 (2010).
 - [21] P. Marek, and R. Filip, Phys. Rev. A **81**, 022302 (2010); J. Fiurášek, Phys. Rev. A **80**, 053822 (2009).
 - [22] T. C. Ralph and A. P. Lund, in *Quantum Communication Measurement and Computing, Proceedings of 9th International Conference*, Ed. A. Lvovsky (AIP, New York 2009).
 - [23] F. Ferreyrol, M. Barbieri, R. Blandino, S. Fossier, R. Tualle-Brouri, and P. Grangieri, Phys. Rev. Lett. **104**, 123603 (2010); F. Ferreyrol, R. Blandino, M. Barbieri, R. Tualle-Brouri, and P. Grangier, Phys. Rev. A **83**, 063801 (2011).
 - [24] M. A. Usuga, R. Ch. Müller, Ch. Wittmann, P. Marek, R. Filip, Ch. Marquardt, G. Leuchs, and U. L. Andersen, Nature Phys **6**, 767 (2010).
 - [25] A. Zavatta, S. Viciani and M. Bellini, Phys. Rev. A **72**, 023820 (2005); A. Zavatta, V. Parigi, M. S. Kim and M. Bellini, New J. Phys. **10**, 123006 (2008).
 - [26] A. Zavatta, V. Parigi, M. S. Kim, H. Jeong and M. Bellini, Phys. Rev. Lett. **103**, 140406 (2009).